

# Wittgenstein versus Tarski: Wittgensteinian interpretations of logic

Terje Aaberge  
Vestlandsforskning

- First Order Language
- Modelling the Domain
- Tarski: Extensional Interpretations
- Wittgenstein: Intensional Interpretations
- Object Language
- Property Language
- Metalanguage

# First Order Language

- Vocabulary
  - Names, Variables, Predicates
  - Logical connectives
- Syntactic rules
- Sentences and formulae
- Rules of deduction
- Logical axioms
- Ontology
  - Axioms
  - Terminological definitions
- Interpretation

# Modelling the Domain

- Extensional models: the domain is modelled as a set consisting of individuals, sets of individuals, sets of ordered pairs of individuals etc.
- Intensional models: the domain is modelled as a directed multi-graph, an individual is then represented by a node and a relation by an arrow connecting the pair of individuals partaking in the relation

# Extensional Interpretation

A name denote an individual, a one-place predicate denotes the set of individuals to which the predicate applies, a two-place predicate the ordered set of pairs of individuals to which the predicate apply etc. This can be symbolised by a map

$$i : V \rightarrow D; i(\text{name}) \mapsto \text{individual}$$
$$i(\text{predicate}) \mapsto \{\text{individuals}\}$$

# Intensional Interpretation

## Object Language

- Measurements
- Operational Definitions
- Observables

# Object Language

- Object language for  $D$ :  $L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$

- Interpretation

$$v: D \rightarrow N \cup N^{(2)}; d \mapsto v(d) = n \quad ; r \mapsto (n_s, n_t) \text{ (isomorphism)}$$

$$\delta: D \rightarrow P_1; d \mapsto \delta(d) = p$$

$$\delta^{(2)}: D \rightarrow P^{(2)}; r \mapsto p^{(2)}$$

- For each observable there exists a unique map defined by the condition of commutativity of the diagrams

$$\begin{array}{ccc}
 N & \xrightarrow{\pi} & P \\
 v \uparrow & \nearrow \delta & \\
 D & & 
 \end{array}
 \quad \pi(v(d)) = \delta(d), \quad \forall d \in D$$

# Truth Conditions

The diagram relates the simulation of measurements determining atomic facts assigning a property to a system  $d$  and the formulation of an atomic sentence expressing such a fact by the juxtaposition  $pn$  of the name  $n$  referring to the system  $d$  and the predicate  $p$  referring to the property, i.e.  $pn$  expresses an atomic fact if  $\pi(n) = p$  for  $n = v(d)$  and  $p = \delta(d)$ ; and similar for relations.

# Abstraction

$$\varepsilon : D \rightarrow E; d \mapsto \varepsilon(d) = e$$

For each observable  $\delta$  there exists a map

$$\rho : E \rightarrow P_1; e \mapsto \rho(e)$$

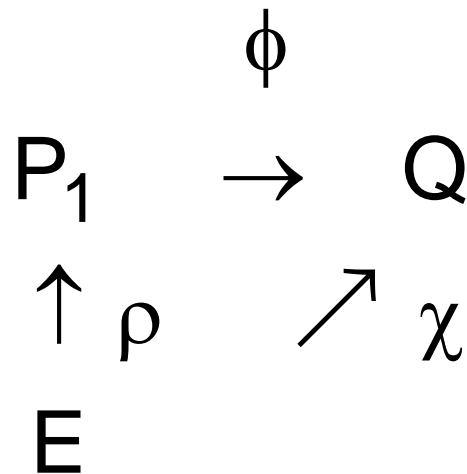
such that

$$\begin{array}{ccc} & P_1 & \\ & \uparrow \rho & \\ D & \nearrow \delta & \\ & \rightarrow E & \\ & \varepsilon & \end{array} \quad \delta(d) = \rho(\varepsilon(d)), \quad \forall d \in D$$



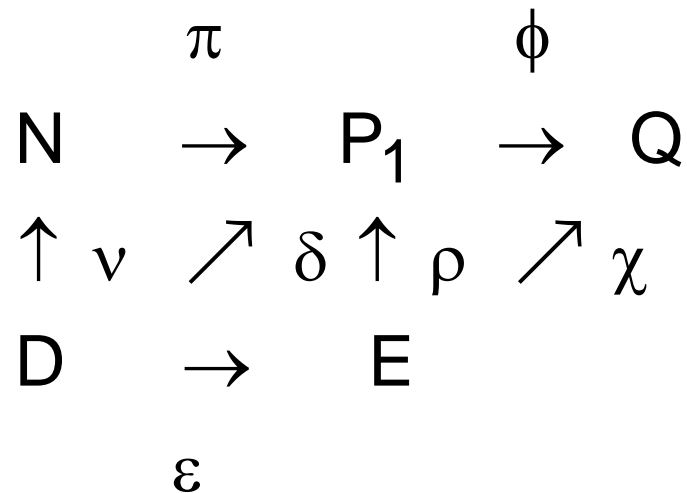
# Property Language

$L(E; P_1 \cup W, Q)$



# Theory

The semantic structure of the theory is described by the diagram:



# Metalanguage

Metalanguage:  $L_G(M_1 \cup M^{(2)}, Q)$

Domain:  $G = D \cup L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$

Names of terms, sentences and formulae

$M_1 = D \cup L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$

Names of relations:  $M^{(2)}$

# Naming Map

$$\eta : G \rightarrow M_1 \cup M^{(2)}; d \mapsto \eta(d) = d$$

$$n \mapsto \eta(n) = n$$

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$$(v(d) = n) \mapsto \eta(v(d) = n) = (d, n)$$

$$(\pi(n) = p) \mapsto \eta(\pi(n) = p) = (n, p)$$

$$\left(\pi(n_s, n_t) = p^{(2)}\right) \mapsto \eta\left(\pi(n_s, n_t) = p^{(2)}\right) = \left((n_s, n_t), p^{(2)}\right)$$

$$(\delta(d) = p) \mapsto \eta(\delta(d) = p) = (d, p)$$

$$\left(\delta^{(2)}(r) = p^{(2)}\right) \mapsto \eta\left(\delta^{(2)}(r) = p^{(2)}\right) = \left(r, p^{(2)}\right)$$

# Observables

$$\alpha: G \rightarrow Q; g \mapsto \alpha(g)$$

$$\begin{array}{ccc} & & \beta \\ & & \\ M_1 \cup M^{(2)} & \rightarrow & Q \\ \eta \uparrow & \nearrow & \alpha \\ & G & \end{array}$$

# Semantic Observable

$$\sigma: G \rightarrow Q; d \mapsto \sigma(d) = D$$

$$r \mapsto \sigma(r) = D^{(2)}$$

$$n \mapsto \sigma(n) = N$$

$$p \mapsto \sigma(p) = P$$

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$$(v(d)=n) \mapsto \sigma(v(d)=n) = P_v$$

$$(\pi(n)=p) \mapsto \sigma(\pi(n)=p) = P_\pi$$

$$\left(\pi(n_s, n_t) = p^{(2)}\right) \mapsto \sigma\left(\pi(n_s, n_t) = p^{(2)}\right) = P_\pi^{(2)}$$

$$(\delta(d)=p) \mapsto \sigma(\delta(d)=p) = P_\delta$$

$$\left(\delta^{(2)}(r) = p^{(2)}\right) \mapsto \sigma\left(\delta^{(2)}(r) = p^{(2)}\right) = P_{\delta^{(2)}}$$

# Informal Interpretation

$Dm$ ,  $m$  is an individual

$Nm$ ,  $m$  is the name of an individual

$Pm$ ,  $m$  is a predicate

$P_{\nu}m_1m_2$ ,  $m_1$  is named  $m_2$

$P_{\delta}m_1m_2$ ,  $m_1$  possesses the property referred to by  $m_2$

# Truth Observable

The truth observable  $\tau$  is given by the values true T, neutral I and false F.  $\tau$  is neutral for all individuals, relations, terms and formulae, and true or false on the sentences, i.e. if  $s$  is a sentence, then the truth of  $s$  is expressed by  $Ts$ .



# Axioms

Axiom 1: the sentences  $P_\pi np$  and  $S_{pn}$  etc. carries the same semantic content

$$P_1 m_1 \wedge N m_2 \Rightarrow P_\pi m_2 m_1 = S m_1 m_2$$

$$P^{(2)} m_1 \wedge N^{(2)} m_2 \Rightarrow P_\pi m_2 m_1 = S m_1 m_2$$

Axiom 2: for each of the commutative diagrams

$$D m_1 \wedge N m_2 \wedge P_1 m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_\delta m_1 m_3 \Rightarrow P_\pi m_2 m_3)$$

$$D^{(2)} m_1 \wedge N^{(2)} m_2 \wedge P^{(2)} m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_{\delta^{(2)}} m_1 m_3 \Rightarrow P_\pi m_2 m_3)$$

Axiom 3: for each of the diagrams the commutativity conditions hold for an atomic sentence iff the sentence is true, i.e.

$$(D m_1 \wedge N m_2 \wedge P_1 m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_\delta m_1 m_3 \Rightarrow P_\pi m_2 m_3)) \Leftrightarrow T m_3 m_2$$

$$(D^{(2)} m_1 \wedge N^{(2)} m_2 \wedge P^{(2)} m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_{\delta^{(2)}} m_1 m_3 \Rightarrow P_\pi m_2 m_3)) \Leftrightarrow T m_3 m_2$$

# Syntactic Rules

Atomic sentence:  $Nn \wedge Pp \Rightarrow Spn$

Conjunction:  $Hf_1 \wedge Hf_2 \Rightarrow H(f_1 \wedge f_2)$

Disjunction:  $Hf_1 \vee Hf_2 \Rightarrow H(f_1 \vee f_2)$

Negation:  $Hf_1 \Rightarrow H\neg f_1$

Univer. quant.:  $Hf(x) \Rightarrow H(\forall_x f(x))$

etc.

# Deduction Rules

Substitution:  $Ss_1 \wedge \forall x \wedge Hf(x) \Rightarrow Sf(s_1)$

Modus ponens:  $Hf_1 \wedge H(f_1 \Rightarrow f_2) \Rightarrow Hf_2$

Generalisation: if it is assumed that the hypotheses underlying the derivation of  $f(x)$  does not depend on  $x$  then

$Hf(x) \Rightarrow T \forall_x f(x)$

# Conclusion

- Intensional interpretations is supported by scientific methodology
- The intensional framework is closed
  - Truth conditions expressed by ontology
  - Operational definitions expressible in the metalanguage
- Tarski: "s" is true if and only if s
  - "Snow is white" is true if and only if snow is white