Wittgenstein versus Tarski: Wittgensteinian interpretations of logic

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- First Order Language
- Modelling the Domain
- Tarski: Extensional Interpretations
- Wittgenstein: Intensional Interpretations
- Object Language
- Property Language
- Metalanguage

First Order Language

- Vocabulary
  - Names, Variables, Predicates
  - Logical connectives
- Syntactic rules
- Sentences and formulae
- Rules of deduction
- Logical axioms
- Ontology
  - Axioms
  - Terminological definitions
- Interpretation

Modelling the Domain

- Extensional models: the domain is modelled as a set consisting of individuals, sets of individuals, sets of ordered pairs of individuals etc.
- Intensional models: the domain is modelled as a directed multi-graph, an individual is then represented by a node and a relation by an arrow connecting the pair of individuals partaking in the relation

Extensional Interpretation

A name denote an individual, a one-place predicate denotes the set of individuals to which the predicate applies, a two-place predicate the ordered set of pairs of individuals to which the predicate apply etc. This can be symbolised by a map

$$i : V \to D; \, i(\text{name}) \mapsto \text{individual}$$

$$i(\text{predicate}) \mapsto \{\text{individuals}\}$$

Intensional Interpretation

Object Language

- Measurements
- Operational Definitions
- Observables

Object Language

- Object language for D: $L_D(N \cup N^{(2)} \cup V,\ P_1 \cup P^{(2)} \cup P_2)$
- Interpretation

$$\nu:\ D \to N \cup N^{(2)};\ d \mapsto \nu(d) = n \quad ;r \mapsto (n_s, n_t)\ (\text{isomorphism})$$

$$\delta:\ D \to P_1;\ d \mapsto \delta(d) = p$$

$$\delta^{(2)}: D \to P^{(2)}; r \mapsto p^{(2)}$$

- For each observable there exists a unique map defined by the condition of commutativity of the diagrams

$$\begin{array}{ccc} & \pi & \\ N & \to & P \\ \nu \uparrow & \nearrow & \delta \\ D & & \end{array} \qquad \pi(\nu(d)) = \delta(d), \quad \forall d \in D$$

Truth Conditions

The diagram relates the simulation of measurements determining atomic facts assigning a property to a system d and the formulation of an atomic sentence expressing such a fact by the juxtaposition pn of the name n referring to the system d and the predicate p referring to the property, i.e. pn expresses an atomic fact if $\pi(n) = p$ for $n = \nu(d)$ and $p = \delta(d)$; and similar for relations.

Abstraction

$$\varepsilon : D \rightarrow E; d \mapsto \varepsilon(d) = e$$

For each observable δ there exists a map

$$\rho : E \rightarrow P_1; \quad e \mapsto \rho(e)$$

such that

$$\begin{array}{ccc} & & P_1 \\ & \nearrow \delta & \uparrow \rho \\ D & \xrightarrow[\varepsilon]{} & E \end{array} \qquad \delta(d) = \rho(\varepsilon(d)), \quad \forall d \in D$$

Property Language

$$L(E;P_1 \cup W,Q)$$

$$\begin{array}{ccc} & \phi & \\ P_1 & \rightarrow & Q \\ \uparrow \rho & & \nearrow \chi \\ E & & \end{array}$$

Theory

The semantic structure of the theory is described by the diagram:

$$
\begin{array}{ccccc}
 & \pi & & \phi & \\
N & \rightarrow & P_1 & \rightarrow & Q \\
\uparrow \nu & \nearrow \delta & \uparrow \rho & \nearrow \chi & \\
D & \rightarrow & E & & \\
 & \varepsilon & & &
\end{array}
$$

Metalanguage

Metalanguage: $L_G(M_1 \cup M^{(2)}, Q)$

Domain: $G = D \cup L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$

Names of terms, sentences and formulae

$M_1 = D \cup L_D(N \cup N^{(2)} \cup V, P_1 \cup P^{(2)} \cup P_2)$

Names of relations: $M^{(2)}$

Naming Map

$$\eta : G \to M_1 \cup M^{(2)}; d \mapsto \eta(d) = d$$

$$n \mapsto \eta(n) = n$$

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•

$$\left(\nu(d) = n\right) \mapsto \eta\left(\nu(d) = n\right) = (d,n)$$

$$\left(\pi(n) = p\right) \mapsto \eta\left(\pi(n) = p\right) = (n,p)$$

$$\left(\pi(n_s,n_t) = p^{(2)}\right) \mapsto \eta\left(\pi(n_s,n_t) = p^{(2)}\right) = \left((n_s,n_t),p^{(2)}\right)$$

$$\left(\delta(d) = p\right) \mapsto \eta\left(\delta(d) = p\right) = (d,p)$$

$$\left(\delta^{(2)}(r) = p^{(2)}\right) \mapsto \eta\left(\delta^{(2)}(r) = p^{(2)}\right) = \left(r,p^{(2)}\right)$$

Observables

$$\alpha : G \to Q;\ g \mapsto \alpha(g)$$

$$\begin{array}{ccc} & & \beta \\ M_1 \cup M^{(2)} & \to & Q \\ \eta \uparrow & & \nearrow \alpha \\ G & & \end{array}$$

Semantic Observable

$$\sigma : G \to Q; d \mapsto \sigma(d) = D$$

$$r \mapsto \sigma(r) = D^{(2)}$$

$$n \mapsto \sigma(n) = N$$

$$p \mapsto \sigma(p) = P$$

•

•

$$(\nu(d) = n) \mapsto \sigma(\nu(d) = n) = P_\nu$$

$$(\pi(n) = p) \mapsto \sigma(\pi(n) = p) = P_\pi$$

$$\left(\pi(n_s, n_t) = p^{(2)}\right) \mapsto \sigma\left(\pi(n_s, n_t) = p^{(2)}\right) = P_\pi^{(2)}$$

$$(\delta(d) = p) \mapsto \sigma(\delta(d) = p) = P_\delta$$

$$\left(\delta^{(2)}(r) = p^{(2)}\right) \mapsto \sigma\left(\delta^{(2)}(r) = p^{(2)}\right) = P_{\delta^{(2)}}$$

Informal Interpretation

Dm, m is an individual

Nm, m is the name of an individual

Pm, m is a predicate

$P_\nu m_1 m_2$, m_1 is named m_2

$P_\delta m_1 m_2$, m_1 posses the property referred to by m_2

Truth Observable

The truth observable τ is given by the values true T, neutral I and false F. τ is neutral for all individuals, relations, terms and formulae, and true or false on the sentences, i.e. if s is a sentence, then the truth of s is expressed by Ts.

Axioms

Axiom 1: the sentences $P_\pi np$ and Spn etc. carries the same semantic content

$$P_1 m_1 \wedge Nm_2 \Rightarrow P_\pi m_2 m_1 = Sm_1 m_2$$

$$P^{(2)} m_1 \wedge N^{(2)} m_2 \Rightarrow P_\pi m_2 m_1 = Sm_1 m_2$$

Axiom 2: for each of the commutative diagrams

$$Dm_1 \wedge Nm_2 \wedge P_1 m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_\delta m_1 m_3 \Rightarrow P_\pi m_2 m_3)$$

$$D^{(2)} m_1 \wedge N^{(2)} m_2 \wedge P^{(2)} m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_{\delta^{(2)}} m_1 m_3 \Rightarrow P_\pi m_2 m_3)$$

Axiom3: for each of the diagrams the commutativity conditions hold for an atomic sentence iff the sentence is true, i.e.

$$\left(Dm_1 \wedge Nm_2 \wedge P_1 m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_\delta m_1 m_3 \Rightarrow P_\pi m_2 m_3)\right) \Leftrightarrow Tm_3 m_2$$

$$\left(D^{(2)} m_1 \wedge N^{(2)} m_2 \wedge P^{(2)} m_3 \Rightarrow (P_\nu m_1 m_2 \wedge P_{\delta^{(2)}} m_1 m_3 \Rightarrow P_\pi m_2 m_3)\right) \Leftrightarrow Tm_3 m_2$$

Syntactic Rules

Atomic sentence: $\text{Nn} \wedge \text{Pp} \Rightarrow \text{Spn}$

Conjunction: $\text{Hf}_1 \wedge \text{Hf}_2 \Rightarrow \text{H}\left(\text{f}_1 \wedge \text{f}_2\right)$

Disjunction: $\text{Hf}_1 \vee \text{Hf}_2 \Rightarrow \text{H}\left(\text{f}_1 \vee \text{f}_2\right)$

Negation: $\text{Hf}_1 \Rightarrow \text{H}\neg\text{f}_1$

Univer. quant.: $\text{Hf}\left(\text{x}\right) \Rightarrow \text{H}\left(\forall_\text{x}\text{f}\left(\text{x}\right)\right)$

etc.

Deduction Rules

Substitution: $Ss_1 \wedge Vx \wedge Hf(x) \Rightarrow Sf(s_1)$

Modus ponens: $Hf_1 \wedge H(f_1 \Rightarrow f_2) \Rightarrow Hf_2$

Generalisation: if it is assumed that the hypotheses underlying the derivation of f(x) does not depend on x then

$Hf(x) \Rightarrow T\ \forall_x f(x)$

Conclusion

- Intensional interpretations is supported by scientific methodology
- The intensional framework is closed
  - Truth conditions expressed by ontology
  - Operational definitions expressible in the metalanguage

- Tarski: ”s” is true if and only if s
  - ”Snow is white” is true if and only if snow is white